

SPECTRAN, a Set of Matlab Programs for Spectral Analysis*

MANUAL

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Abstract

*Spectral analysis is one of the most important areas of time series econometrics. The use of spectral measures is widespread in different science fields such as economics, physics, engineering, geology. The SPECTRAN toolbox has been developed to facilitate the application of spectral concepts to univariate as well as to multivariate series. It offers a variety of frequency-domain techniques and supports the statistical inference. It also provides convenient tools for the examination of the results, e.g. functions for writing the output to a file or functions specially designed for plotting the estimated spectral measures. The key feature of SPECTRAN is the user-friendliness embodied in, e.g., the central function **spectran** which performs the whole analysis with default settings, but also gives the user the possibility to adjust them. This document sets out the most relevant spectral concepts and their implementation in SPECTRAN. Finally, three examples shall illustrate the application of different toolbox function to macroeconomic data.*

JEL Classification: C18, C22, C32, C87

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*This document has been written based on the release spectran2012. In case of possible toolbox changes, the manual will be updated accordingly. Please note that each online release of the toolbox is published with the recent manual version.

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1 Covariances and Correlations

Even though SPECTRAN is a toolbox developed for spectral analysis, before different spectral concepts are introduced and their implementation is described, a little attention should be given to the estimation of covariances and correlations. As will become clear later in this document, these quantities play an important role in deriving spectral measures. Both auto- and cross-covariances are estimated in SPECTRAN by the function **crosscov**. For the computation of the autocovariances of a series x_t ($t = 1, \dots, N$) **crosscov** applies the formula

$$\hat{\gamma}_x(j) = \frac{1}{N} \sum_{t=j+1}^N (x_t - \bar{x})(x_{t-j} - \bar{x}), \quad (1)$$

where $\hat{\gamma}_x(j)$ is the sample autocovariance of x_t at the lag j and \bar{x} is the estimated mean value of x_t given by

$$\bar{x} = \frac{1}{N} \sum_{t=1}^N x_t$$

In the case of the cross-covariances between two series x_t and y_t ($t = 1, \dots, N$), the following formula is used:

$$\hat{\gamma}_{xy}(j) = \frac{1}{N} \sum_{t=j+1}^N (x_t - \bar{x})(y_{t-j} - \bar{y}) \quad (2)$$

The function **crosscor** calculates the sample auto- and cross-correlations based on eq. (1) and eq. (2), respectively.

The user may want to study the sample covariances or correlations prior to the spectral analysis. To this end, he can use the function **corwrite** which writes the sample covariances or correlations up to a specified lag to a text file.

```
function corwrite(fid,ns,cr,corlag,vnames)
%*****
%
%      This function writes the autocorrelations or
%      cross correlations in a text file
%
%      INPUTS:
%      -----
%      fid : id of the text file where the output is to be written
%      ns  : number of series (1 or 2)
%      cr   : autocorrelations or cross correlations
%      corlag : number of leads and lags at which the
%              auto/cross correlations are computed;
%      vnames : name/s of the series
```

2 Univariate Spectral Analysis

2.1 Spectrum Estimation

The spectrum of a time series constitutes the central concept of spectral analysis. It enables the identification of frequency components that make the greatest contribution to the overall variance of the series at hand. The spectrum is given as the Fourier transform of the autocovariance function (Priestley, 1981, p. 213):

$$f_x(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \gamma_x(\tau) e^{-i\omega\tau} d\tau, \quad (3)$$

where ω denotes the angular frequency, $f_x(\omega)$ denotes the spectrum of x_t and $\gamma_x(\tau)$ is the autocovariance function of x_t . In the case of discrete data, eq. (3) becomes:

$$\begin{aligned} f_x(\omega) &= \frac{1}{2\pi} \sum_{j=-\infty}^{\infty} \gamma_x(j) e^{-i\omega j} \\ &= \frac{1}{2\pi} \sum_{j=-\infty}^{\infty} \gamma_x(j) \cos(\omega j) \end{aligned} \quad (4)$$

Since the data usually are finite and discrete in practice, the sample counterpart of the theoretical spectrum, called periodogram, can be seen as a truncated version of the theoretical spectrum with theoretical covariances replaced by their estimates. The periodogram is an unbiased but inconsistent spectrum estimate. In order to reduce its variance, the so-called smoothing is applied. In the time domain, it reduces to the application of a window function to the sample autocovariance function (Priestley, 1981, p. 502):

$$\hat{f}_x(\omega) = \frac{1}{2\pi} \sum_{j=-(N-1)}^{N-1} w_m(j) \hat{\gamma}_x(j) \cos(\omega j), \quad (5)$$

where $\hat{f}_x(\omega)$ denotes the estimate of $f_x(\omega)$, $\hat{\gamma}_x(j)$ is given by eq. (1) and $w_m(j)$ represents a window function. Function $w_m(j)$ satisfies the following properties (Koopmans, 1974, p. 266):

- i) $0 \leq w_m(j) \leq w_m(0) = 1$,
- ii) $w_m(j) = w_m(-j)$ for all k ,
- iii) $w_m(j) = 0$ for all $|j| > m$

where $m < N-1$ is an integer called truncation point or lag number. In SPECTRAN, three window function types are available: the Blackman–Tukey window (or simply Tukey window), the Parzen window and the Tukey–Hanning window. In addition, confidence intervals for the smoothed spectrum estimates can be computed as described in Koopmans (1974, p. 274). The function **periodg** is used in SPECTRAN for spectrum estimation.

```

function sp = periodg(x,win,winlag,alpha)
%*****
%       This function computes the (smoothed) periodogram
%
%       INPUTS:
%       -----
%       REQUIRED
%           x : series
%
%       OPTIONAL
%           win : window used for smoothing the periodogram;
%               = 0 : no smoothing is performed
%               = 1 : Blackman–Tukey window
%               = 2 : Parzen window
%               = 3 : Tukey–Hanning window
%               Parzen window is used, if win is not input to periodg or
%               if win is empty,
%           winlag : window lag size; if it is empty, winlag is computed by the program
%           alpha : significance level needed for calculaction of the confidence
%                  intervals; if alpha is not input to periodg or if alpha is
%                  empty, confidence intervals are not computed
%
%       OUTPUTS:
%       -----
%           sp : structure with the following fields:
%               .f      : (smoothed) periodogram
%               .n      : length of x
%               .win     : window used for smoothing the periodogram
%               .winlag  : window lag size; empty if no smoothing was
%                       performed
%               .a       : parameter of the Blackman–Tukey window; a is field
%                       of sp if win = 1
%               Additional field, if alpha is input to periodg and is not
%               empty
%               .fconf   : confidence intervals for f

```

If the computation of confidence intervals for the estimated spectrum is desired, the user can either pass the value for the significance level (*alpha*) to **periodg** or he can leave this argument unspecified and, instead, use the function **spconf** after calling **periodg**.

```

function fconf = spconf(sp,alpha)
%*****
%       This function computes the confidence intervals for the smoothed
%       periodogram
%       See Koopmans, L.H.(1974), "The Spectral Analysis of Time Series",
%       p.274, 279
%

```

```

%      INPUTS:
%-----
%      REQUIRED
%      sp : structure , output of function periodg
%
%      OPTIONAL
%      alpha : significance level needed for calculation of
%              the confidence intervals;
%              alpha = 0.05, if alpha is not input to spconf or
%              if alpha is empty
%
%      OUTPUT:
%-----
%      fconf : confidence intervals for the smoothed periodogram

```

2.2 Output Files and Plots

It may be convenient to save the estimation output in a text file. In this way, the user can have access to the results at any time. In SPECTRAN, the function **specwrite** is designed to write the output of the spectral analysis to a text file. Function **specwrite** let the user decide for which frequency band the results are to be displayed.

```

function specst = specwrite(fid,per,sp,fband,mph,mphconf,phfband,vnames,ppi)
%*****
% This function writes results of the spectral analysis to a text file
%
%      INPUTS:
%-----
%      fid : id of the text file where the output is to be written
%      per : frequency of the data (number of periods per year)
%      sp : structure being output of the function periodg, crossspan or
%           mulparspan
%      fband : frequency interval for which results are displayed;
%              default is [0,pi]
%      mph : mean phase angle;
%              it should be empty if sp is output of periodg
%      mphconf : confidence interval for mph
%              it should be empty if sp is output of periodg
%      phfband : frequency interval for which results for the mean phase angle
%                are displayed; default is [0,pi]
%              it should be empty if sp is output of periodg
%      vnames : name/s of the series; it can be empty
%      ppi : 0, do not express angular measures in terms of pi (default)
%            1, express all angular measures in terms of pi
%
%      OUTPUT: (optional)
%-----
%      specst: structure depending on sp; all fields refer to the
%              interval given by fband

```

```

%
%      All possible fields , if sp is output of periodg:
% .frq      : frequencies
% .f        : spectrum of the series
% .fconf    : confidence interval
%
%      All possible fields , if sp is output of crosspan:
% .frq      : frequencies
% .fx       : (smoothed) periodogram of the reference series
% .fy       : (smoothed) periodograms of all other series
% .co       : matrix with columns containing coherency
%           : between the ref. series and a particular series
% .sco      : matrix with columns containing coherence
%           : (squared coherency) between the ref. series
%           : and a particular series
% .ga       : matrix with columns containing gain
%           : between the ref. series and other series
% .ph       : matrix with columns containing phase angle
%           : between the ref. series and other series
% .pht      : matrix with columns containing phase delay
%           : of a particular series relative to the ref. series
% .phd      : matrix with columns containing group delay
%           : of a particular series relative to the ref. series
% .mph      : vector with mean phase angle values
%           : between the ref. series and other series
% .fxconf   : confidence intervals for fx
% .fyconf   : confidence intervals for fy
% .cconf    : confidence intervals for co
% .gconf    : confidence intervals for ga
% .pconf    : confidence intervals for ph
% .mphconf  : confidence intervals for mph
%
%      All possible fields , if sp is output of mulparspan:
% .frq      : frequencies
% .fx       : (smoothed) periodogram of the reference series
% .fy       : (smoothed) periodograms of all other series
% .mco      : matrix with columns containing multiple coherency
%           : between the ref. series and other series
% .msco     : matrix with columns containing multiple coherence
%           : (multiple squared coherency) between the ref. series
%           : and a particular series
% .pco      : matrix with columns containing partial coherency
%           : between the ref. series and other series
% .psco     : matrix with columns containing partial coherence
%           : (partial squared coherency) between the ref. series
%           : and a particular series
% .pga      : matrix with columns containing partial gain
%           : between the ref. series and a particular series
% .pph      : matrix with columns containing partial phase angle
%           : between the ref. series and a particular series
% .ppht     : matrix with columns containing partial phase delay
%           : of a particular series relative to the ref. series

```

```

%          .pphd      : matrix with columns containing partial group delay
%                      of a particular series relative to the ref. series
%          .mpph      : vector with mean phase angle values
%                      between the ref. series and a particular series
%          .fxconf     : confidence intervals for fx
%          .fyconf     : confidence intervals for fy
%          .mpphconf: confidence intervals for mpph

```

In addition to the written output, the user may want to study the graphical representation of the estimated spectra. In SPECTRAN, the function **specplot** is designed for plotting, e.g., the estimated spectra along with their confidence bands. Moreover, function **specplot** also provides a feature, particularly interesting for business cycle analysts. If the user does not restrict the frequency band to the band exactly covering the frequencies corresponding to the business cycle periodicities, **specplot** highlights the results in the business cycle frequency band by separating them with lines from the results in the remaining region.

```

function specplot(frq,per,mul,spec,fband,speconf,alpha,specname,vnames)
%*****
%          This function plots (non-angular) spectral measures and their
%          confidence intervals
%
%          INPUTS:
%          -----
%          frq : frequencies
%          per : frequency of the data (number of periods per year)
%          mul : 0, if univariate spectral measures are to be plotted
%                1, if multivariate spectral measures are to be plotted
%          spec : matrix with uni- or multivariate spectral measures with
%                columns corresponding to a particular series or
%                a pair of series
%          fband : frequency interval for which results are displayed;
%                default is [0,pi]
%          speconf : confidence intervals for spec; it can be empty
%          alpha : significance level; it can be empty
%          specname : name of the spectral measure; it can be empty
%          vnames : name/s of the series; it can be empty

```

3 Multivariate Spectral Analysis

3.1 Bivariate Spectral Measures

Analogously to the spectrum in the univariate case, the cross-spectrum is the central concept in the analysis of multivariate time series. The cross-spectrum $f_{xy}(\omega)$ between two time series x_t and y_t is given by the Fourier transform of the covariance function $\gamma_{xy}(\tau)$. As it is complex-valued, it can be decomposed into the real part

$c_{xy}(\omega)$ (cospectrum) and the imaginary part $q_{xy}(\omega)$ (quadrature spectrum) which in the case of discrete data are given by the following formulas (Priestley, 1981, p. 659):

$$c_{xy}(\omega) = \frac{1}{2\pi} \sum_{j=-\infty}^{\infty} \gamma_{xy}(j) \cos(\omega j)$$

$$q_{xy}(\omega) = -\frac{1}{2\pi} \sum_{j=-\infty}^{\infty} \gamma_{xy}(j) \sin(\omega j)$$

In SPECTRAN, the estimates of the cospectrum and the quadrature spectrum are computed by the function **cospqu** in the following way:

$$\hat{c}_{xy}(\omega) = \frac{1}{2\pi} \sum_{j=-(N-1)}^{N-1} w_m(j) \hat{\gamma}_{xy}(j) \cos(\omega j) \quad (6)$$

$$\hat{q}_{xy}(\omega) = -\frac{1}{2\pi} \sum_{j=-(N-1)}^{N-1} w_m(j) \hat{\gamma}_{xy}(j) \sin(\omega j), \quad (7)$$

where in the case of smoothed quantities $w_m(j)$ represents a window function. If raw quantities should be obtained, $w_m(j)$ can be set to one for $|j| < m = N - 1$ and to zero otherwise.

```
function cqsp = cospqu(x,y,win,winlag)
%*****
%      This function computes the (smoothed) cross-periodogram
%
%      INPUTS:
%      -----
%      REQUIRED
%          x,y : series
%
%      OPTIONAL
%          win : window used for smoothing the periodogram;
%                = 0 : no smoothing is performed
%                = 1 : Blackman-Tukey window
%                = 2 : Parzen window
%                = 3 : Tukey-Hanning window
%                Parzen window is used, if win is not input to cospqu or
%                if win is empty,
%          winlag : window lag size; if it is not input to cospqu or if it
%                   is empty, winlag is computed by the program
%
%      OUTPUTS:
%      -----
%          cqsp : structure with the following fields:
%                .c      : cospectrum
%                .q      : quadrature spectrum
```

```

%           .n      : length of x and y
%           .win    : window used for smoothing the periodogram
%           .winlag  : window lag size; empty if no smoothing was
%                     performed
%           .a      : parameter of the Blackman–Tukey window;
%                     a is field of sp if win = 1

```

To measure comovements between time series, several concepts based on the cospectrum and the quadrature spectrum have been proposed in the literature. The most commonly used are coherency, squared coherency, phase angle and gain; see, e.g., [Koopmans \(1974, pp. 135\)](#). Coherency is defined as:

$$R_{xy}(\omega) = \sqrt{\frac{c_{xy}^2(\omega) + q_{xy}^2(\omega)}{f_x(\omega)f_y(\omega)}} \quad (8)$$

whereas coherence $R_{xy}^2(\omega)$ is the squared value of $R_{xy}(\omega)$. Their values can vary between zero and one and measure the strength of the relationship between the considered series at each frequency. Another very useful cross-spectral measure, the phase angle, is defined as follows:

$$\phi_{xy}(\omega) = \arctan \left[\frac{q_{xy}(\omega)}{c_{xy}(\omega)} \right] \quad (9)$$

The phase provides information on the lead-lag relation between two series at each frequency. This aspect will be addressed in more detail in the next subsection. Moreover, by the examination of the phase angle values one can also identify a positive or negative relationship between two series at each frequency. A comprehensive discussion of the phase angle can be found in, e.g., [Marczak and Beissinger \(2012\)](#). Gain is referred to as the gain function of the filter which transforms x_t into the best approximation to y_t in the mean square sense. In other words, it represents a function which changes the amplitude of each frequency component of x_t to obtain the best approximation to y_t . The formal definition of the gain is given by:

$$B_{xy}(\omega) = \frac{\sqrt{c_{xy}^2(\omega) + q_{xy}^2(\omega)}}{f_x(\omega)} \quad (10)$$

Estimates of the comovement indicators above are obtained by substituting $c_{xy}(\omega)$, $q_{xy}(\omega)$, $f_x(\omega)$ and $f_y(\omega)$ in eq. (8), (9) and (10) with the quantities computed according to eq. (5), (6) and (7). This procedure has been implemented in SPECTRAN in the function **cohepha**.

```

function cgpsp = cohepha(cqsp, fxx, fyy)
%*****
%       This function computes coherency, coherence (squared coherency),
%       gain and phase angle

```

```

%
%   INPUTS:
%-----
%       cqsp : structure , output of function cospqu
%       fxx  : (smoothed) periodogram of x
%       fyy  : (smoothed) periodogram of x
%
%
%   OUTPUTS:
%-----
%       cgpsp : structure with the following fields:
%       .co    : coherency
%       .sco   : coherence (squared coherency)
%       .ga    : gain
%       .ph    : phase angle
%       .n     : length of x and y
%       .win   : window function
%       .winlag : window lag size

```

Confidence intervals for the coherency, gain and the phase angle can be computed as described in [Koopmans \(1974, pp. 282–287\)](#) by the function **cgpcnf**.

```

function [cconf,gconf,pconf] = cgpcnf(cgpsp,fx,fy,alpha)
%*****
%   This function computes the confidence interval for the coherency,
%   gain and phase angle
%   See Koopmans, L.H.(1974), "The Spectral Analysis of Time Series",
%   pp. 282–287, Table 8.1 (p.279)
%
%   INPUTS:
%-----
%   REQUIRED
%       cgpsp : structure , output of function cohepha
%       fx    : smoothed periodogram of x
%       fy    : smoothed periodogram of y
%
%   OPTIONAL
%       alpha : significance level needed for calculation of
%               the confidence intervals;
%               alpha = 0.05, if alpha is not input to cgpsp or
%               if alpha is empty
%
%   OUTPUTS:
%-----
%       cconf : confidence intervals for the coherency
%       gconf : confidence intervals for the gain
%       pconf : confidence intervals for the phase angle

```

The user may want to directly compute spectra, coherency, gain and phase angle, possibly also with the respective confidence intervals, all at once. This can be accomplished by using the function **crosspan**. Function **crosspan** additionally calculates

the so-called phase delay and the group delay. The phase delay, $\phi_{t,xy}(\omega) = \phi_{xy}(\omega)/\omega$, expresses the shift of each frequency component of one series relative to the corresponding component of the other series in terms of time units depending on the data periodicity. The group delay, $\phi_{g,xy}(\omega) = d\phi_{xy}(\omega)/d\omega$, measures the change of the shift of both frequency components relative to each other. Calculation of both measures is based on the estimated phase angle, $\hat{\phi}_{xy}(\omega)$.

```

function crossp = crossspan(y,win,winlag,alpha)
%*****
%   This function performs the cross-spectral analysis and optionally
%   calculates the confidence intervals for the estimated spectra of
%   two time series , coherency , gain and phase angle
%
%
%   INPUTS:
%-----
%   REQUIRED
%       y : (ly x ny) matrix with the series ;
%           the program assumes that the first column contains
%           the reference series
%
%   OPTIONAL
%       win : window used for smoothing the periodogram ;
%            = 0 : no smoothing is performed
%            = 1 : Blackman-Tukey window
%            = 2 : Parzen window
%            = 3 : Tukey-Hanning window
%            Parzen window is used , if win is not input to crossspan or
%            if win is empty ,
%       winlag : window lag size ; if it is not input to crossspan or if it
%            empty , winlag is computed by the program
%       alpha : significance level needed for calculation of the confidence
%            intervals ; if alpha is not input to crossspan or if alpha is
%            empty , confidence intervals are not computed
%
%
%   OUTPUT:
%-----
%       crossp : structure with the following fields :
%       .freq   : frequencies
%       .fx      : (smoothed) periodogram of the reference series
%       .fy      : (smoothed) periodograms of all other series
%       .co      : matrix with columns containing coherency
%                 between the ref. series and a particular series
%       .sco     : matrix with columns containing coherence
%                 (squared coherency) between the ref. series
%                 and a particular series
%       .ga      : matrix with columns containing gain
%                 between the ref. series and a particular series
%       .ph      : matrix with columns containing phase angle
%                 between the ref. series and a particular series

```

```

%      .pht      : matrix with columns containing phase delay
%                  of a particular series relative to the ref. series
%      .phd      : matrix with columns containing group delay
%                  of a particular series relative to the ref. series
%      .n        : length of the series
%      Additional fields , if alpha is input to crossspan and is not
%      empty:
%      .fxconf   : confidence intervals for fx
%      .fyconf   : confidence intervals for fy
%      .cconf    : confidence intervals for co
%      .gconf    : confidence intervals for ga
%      .pconf    : confidence intervals for ph

```

3.2 Multiple Coherence and Partial Spectral Measures

So far, the multivariate concepts and their implementation were restricted to the bivariate case. If more than two series are involved, the overall relationship between a reference series and the other series, in following referred to as inputs, can be assessed. Moreover, to determine the relationship between a reference series and an input series, one should take account of the interactions between this input series and the remaining ones. These considerations imply that in a multivariate situation with more than two variables, concepts are needed that imitate the ideas well known from the multivariate linear regression theory. The formal representation of the concepts reviewed in this subsection primarily follows [Jenkins and Watts \(1968, pp. 487–490\)](#).

The measure that can be considered as a frequency domain counterpart of the coefficient of determination is called multiple coherence and measures the proportion of the reference series spectrum that can be explained by the input series. The multiple coherence between the series x_{1t} and the inputs $x_{2t,3t,\dots,pt}$ is defined as:

$$R_{1.2,3,\dots,p}^2(\omega) = 1 - \frac{\det F(\omega)}{F_{11}(\omega)M_{11}(\omega)}, \quad (11)$$

where $F(\omega)$ is the cross-spectral matrix. Its diagonal elements $f_{ii}(\omega)$ ($i = 1, \dots, p$) represent spectra of the corresponding series whereas the off-diagonal elements $f_{ij}(\omega)$ ($i, j = 1, \dots, p$ and $i \neq j$) represent cross-spectra between series x_i and x_j and have the property $f_{ij}(\omega) = f_{ji}^*(\omega)$, where “*” denotes the complex conjugate. $F_{ij}(\omega)$ denotes the (ij) -th element of $F(\omega)$ ($i, j = 1, \dots, p$), $\det F(\omega)$ is the determinant of $F(\omega)$ and $M_{ij}(\omega)$ is the (ij) -th minor of $F(\omega)$ ($i, j = 1, \dots, p$). The multiple coherency $R_{1.2,\dots,p}(\omega)$ is given as the positive square root of $R_{1.2,\dots,p}^2(\omega)$.

In analogy to the partial correlation coefficient, the partial coherence or its square root, partial coherency, measures the cet. par. relationship between the reference series and an input series at each frequency. The partial coherence between x_{1t} and x_{jt} ($1 < j \leq p$) is given by:

$$r_{1j|q(j)}^2(\omega) = \frac{|M_{j1}(\omega)|^2}{M_{11}(\omega)M_{jj}(\omega)}, \quad (12)$$

where $q(j)$ is a set of indices such that $q(j) = \{2, \dots, p\} \setminus j$.

It is also possible to quantify the so-called partial phase angle between x_{1t} and x_{jt} , i.e the phase difference between x_{1t} and x_{jt} after allowing for the phase differences between all inputs and between x_{1t} and x_{kt} ($k \in q(j)$). Formally, it can be described as follows:

$$\phi_{1j|q(j)}(\omega) = \arctan \left[\frac{\Im(M_{j1}(\omega))}{\Re(M_{j1}(\omega))} \right], \quad (13)$$

where $\Re(\cdot)$ and $\Im(\cdot)$ denote the real part and the imaginary part of a complex number, respectively. Strongly related to the partial phase angle are the concepts of the partial phase delay, $\phi_{t,1j|q(j)}(\omega) = \phi_{1j|q(j)}(\omega)/\omega$, and the partial group delay, $\phi_{g,1j|q(j)}(\omega) = d\phi_{1j|q(j)}(\omega)/d\omega$.

Similarly to the concept of the gain function introduced in the bivariate case, the partial gain between x_{1t} and x_{jt} is defined as:

$$B_{1j|q(j)}(\omega) = \frac{|M_{j1}(\omega)|}{F_{11}(\omega)} \quad (14)$$

The estimates of the multiple coherence (coherency), partial coherence (coherency), partial phase angle and partial gain can be obtained in the following way. First, elements of the cross-spectral matrix $F(\omega)$ are replaced by their sample counterparts, thereby yielding the estimate of $F(\omega)$, $\hat{F}(\omega)$. Then, eq. (11), (12), (13) and (14) are applied by substituting values derived from $F(\omega)$ with those derived from $\hat{F}(\omega)$. In SPECTRAN, this procedure is implemented in the function **mulparspan**.

```
function mulsp = mulparspan(y,win,winlag,alpha)
%*****
%      This function computes multiple (squared), partial (squared)
%      coherency, partial phase angle and partial gain between
%      the reference series and other series
%
%      INPUTS:
%      -----
%      REQUIRED
%      y : (ly x ny) matrix with the series;
%           the program assumes that the first column contains
%           the reference series
%
%      OPTIONAL
%      win : window used for smoothing the periodogram;
%            = 0 : no smoothing is performed
%            = 1 : Blackman–Tukey window
%            = 2 : Parzen window
%            = 3 : Tukey–Hanning window
%            Parzen window is used, if win is not input to crossspan or
%            if win is empty,
%      winlag : window lag size; if it is not input to crossspan or if it
%               empty, winlag is computed by the program
```

```

%      alpha : significance level needed for calculation of the confidence
%              intervals; if alpha is not input to crossspan or if alpha is
%              empty, confidence intervals are not computed
%
%      OUTPUT:
%      -----
%      mulsp : structure with following fields:
%      .frq      : frequencies
%      .fx        : (smoothed) periodogram of the reference series
%      .fy        : (smoothed) periodograms of all other series
%      .mco       : matrix with columns containing multiple coherency
%                  between the ref. series and a particular series
%      .msco      : matrix with columns containing multiple
%                  coherence (squared multiple coherency) between
%                  the ref. series and a particular series
%      .pco       : matrix with columns containing partial coherency
%                  between the ref. series and a particular series
%      .psco      : matrix with columns containing partial
%                  coherence (squared partial coherency) between
%                  the ref. series and a particular series
%      .pga       : matrix with columns containing partial gain
%                  between the ref. series and a particular series
%      .pph       : matrix with columns containing partial phase angle
%                  between the ref. series and a particular series
%      .ppht      : matrix with columns containing partial phase delay
%                  of a particular series relative to the ref. series
%      .pphd      : matrix with columns containing partial group delay
%                  of a particular series relative to the ref. series
%      .n         : length of the series
%      Additional fields, if alpha is input to mulparspan and is not
%      empty:
%      .fxconf    : confidence intervals for fx
%      .fyconf    : confidence intervals for fy

```

3.3 Mean (Partial) Phase Angle

Sometimes, if mean direction of the relationship between time series in a certain frequency band is of interest, it seems natural to compute mean values of some co-movement measure in this frequency band. However, in the case of the phase angle, building the standard arithmetic mean fails to be the appropriate procedure. Due to circular nature of the phase angle, one should compute the mean according to the concept specially devoted to the data measured on the angular scale; see, e.g., [Zar \(1999\)](#). In SPECTRAN, this can be achieved by using the function **meanphconf** which additionally returns confidence intervals for the mean values. The user can supply either the values of the ordinary phase angle obtained with **crosspan** (or **cohepha**) or the values of the partial phase angle computed with **mulparspan**. Calculations made by **meanphconf** are based on functions provided by [Berens \(2009\)](#).

```

function [mph,mphconf] = meanphconf(ph,frq ,pband)
%*****
% This function computes the mean (partial) phase angle values and
% their confidence intervals.
% Computation is based on the circular statistics.
% This function uses functions circ_mean and cir_confmean from the toolbox
% CircStats2011f by Philipp Berens.
%
% INPUTS:
%-----
% REQUIRED
%     ph : matrix with columns containing (partial) phase angle
%           corresponding to the ref. series and a particular series
%     frq : frequencies , frq and ph must be of the same size;
%           the program assumes that the rows of frq and ph correspond to
%           each other
%
% OPTIONAL, if the (partial) phase angle values are to be averaged
% over a specified interval
%     pband : time interval expressed in time units;
%             (partial) phase angle values are averaged over pband
%
% OUTPUTS:
%-----
%     mph : mean (partial) phase angle values
%     mphconf : confidence intervals for mph

```

3.4 Output Files and Plots

The output of the multivariate spectral analysis described in Section 3.1 or 3.2 can be written to a text file by the function **specwrite** (see the description in Section 2.2). To this end, the user has to supply the structure being output of **crossspan** or **multiparspan**, respectively. Optionally, the mean (partial) phase angle values and their confidence intervals, both computed with **meanphconf** can also be written if *mph* and *mphconf* are passed to the function. Similarly to the univariate case, the function **specplot** is as well capable of plotting most of the multivariate spectral measures (see Section 2.2). As for the (partial) phase angle, the function **phplot** is more suitable since it depicts phase angle values in form of dots and, if desired, the confidence intervals for the estimates as lines. This allows to illustrate the results for each frequency on a linear scale as if it were a “straightened” circular scale.

```

function phplot(frq ,per ,ph ,fband ,phconf ,alpha ,vnames ,mulvar)
%*****
% This function plots the (partial) phase angle values with
% their confidence intervals.
%

```



```

%      INPUTS:
%-----
%      frq : frequencies
%      per : frequency of the data (number of periods per year)
%      ph  : matrix with phase angle values with columns
%            corresponding to the reference series and
%            a particular series
%      fband : frequency interval for which results are displayed;
%            default is [0,pi]
%      phconf : confidence intervals for ph; it can be empty
%      alpha : significance level; it can be empty
%      vnames : name/s of the series; it can be empty
%      mulvar : 0, ordinary phase angle (default)
%              1, partial phase angle

```

3.5 Lead-Lag Analysis

One of the tasks of the SPECTRAN user, e.g. a business cycle analyst, may be to establish the lead-lag relation between two time series. As already mentioned in Section 3.1, the concept of the phase angle plays a major role in this context. One can draw on either the ordinary phase angle given by eq. (9) or the partial phase angle given by eq. (13). The interpretation of each (partial) phase angle value may, however, be somewhat cumbersome. To simplify the identification of the lead-lag pattern, SPECTRAN proposes two strategies. The first one relies on finding that frequency which corresponds to the strongest relationship between the series, here represented by the highest (partial) coherence. The (partial) phase angle value at this frequency serves then as an indicator for the leading or lagging behavior. The second possibility is to consider the (partial) phase angle values averaged over certain frequencies. The function **leadlagan** designed in SPECTRAN for the lead-lag analysis takes account of these two possibilities. To take advantage of them by calling **leadlagan**, the user has to provide the structure being the output of **crossspan** or **mulparspan** depending on whether he wants to pin down the analysis to the ordinary or the partial phase angle, respectively. Optionally, the user can also pass the mean (partial) phase angle values along with their confidence intervals and the frequency band over which the (partial) phase angle values have been averaged. The output of **leadlagan** is written to a text file.

```

function llst = leadlagan(fid,per,sp,fband,mph,mphconf,phfband,vnames,ppi)
%*****
%                                LEAD-LAG ANALYSIS
%
%      Phase shifts between the reference series and an other series
%      at the most important frequency are computed.
%      The most important frequency is defined as the one at which the
%      highest (partial) coherence occurs; (partial) coherence is used as
%      a measure of the relationship between two cycles.

```

```

%
%
%   INPUTS:
%-----
%   fid : id of the text file where the output is to be written
%   per : frequency of the data (number of periods per year)
%   sp  : structure being output of crossspan or mulparspan
%   fband : frequency interval for which results are displayed;
%          default is [0,pi]
%   mph  : mean (partial) phase angle; it can be empty
%   mphconf : confidence interval for mph; it should be empty if mph is
%            empty
%   phfband : frequency interval for which results for the mean (partial)
%            phase angle are displayed; default is [0,pi]
%   vnames : name/s of the series
%   ppi : 0, do not express angular measures in terms of pi (default)
%        1, express all angular measures in terms of pi
%
%   OUTPUT: (optional)
%-----
%   llst : structure with following fields:
%           .masco      : maximum (partial) coherences
%           .maxfrq     : frequencies corresponding to maxsco
%           .maxph      : (partial) phase angle values corresponding
%                        to maxsco
%           .mapht      : (partial) phase delay values corresponding
%                        to maxsco
%           .maxphconf  : confidence intervals for maxph;
%                        field if pconf is field of sp

```

4 Function *spectran*

SPECTRAN is a toolbox which tries to bring in line two important properties. One of them is its flexibility which allows the user to customize his analysis by picking functions yielding the most desired output only. The other one is the user-friendliness manifested by the possibility of obtaining quite a large output in a few steps. In light of the idea of the user-friendliness, the function **spectran** has been designed. This function combines all building blocks described in the previous sections concerning computational aspects and output representation. The function **spectran** offers the user a choice of numerous options. As regards the methodology, he can decide whether in case of more than two series standard bivariate measures or partial measures are to be computed, whether the lead-lag analysis should be performed etc. As for the output files, it is left up to the user whether the text file should be written, whether the results should be plotted, whether the confidence intervals should be plotted, for which periodicity band the results should be displayed etc. With an appropriate configuration, the user can thus arrive at the desired function output in one step

instead of calling several functions.

```

function spectr = spectran(y,per,varargin)
%*****
%                               SPECTRAL ANALYSIS
%
%   This program computes spectrum, (multiple/partial) coherence,
%   (partial) phase angle, (partial) phase delay, (partial) gain and
%   auto- or cross-correlations between a reference series and other series.
%
%                               The syntax :
%   spectr = spectran(y,per,'option1',optionvalue1,'option2',optionvalue2,...)
%
%
%   INPUTS :
%   -----
%   REQUIRED
%       y : (ly x ny) matrix with the series;
%           if ny = 1, univariate spectral analysis and computation
%           of autocorrelations of y are performed,
%           if ny > 1, bivariate or multivariate spectral analysis
%           and computation of cross-correlations are performed;
%           the program assumes that the first column contains
%           the reference series
%       per : frequency of the data (number of periods per year)
%   -----
%   OPTIONS
%       'vnames' : string array with names for the series; the program
%                   assumes that their order coincides with the order in y;
%                   default: refseries, series1, series2, ...
%       'corlag' : number of leads and lags at which the
%                   auto-/cross-correlations are computed; default: ly-1
%       'win' : window function used for (cross-)periodogram smoothing
%               0, no window is applied (nonsmoothed periodogram).
%               1, the Blackman-Tukey window
%               2, the Parzen window (default)
%               3, the Tukey-Hanning window
%       'winlag' : window lag size;
%                   default: depending on the window function
%       'mulvar' : 0, if bivariate analysis in case of ny > 2
%                   is to be performed (default);
%                   (always 0 for ny <= 2)
%                   1, if multivariate analysis in case of ny > 2
%                   is to be performed
%       'conf' : 0, do not compute confidence intervals for the spectral
%                   measures (default)
%                   1, compute confidence intervals
%       'alpha' : significance level; default (if conf = 1): 0.05
%       'pband' : time interval expressed in time units corresponding to
%                   per, results are displayed and saved for the chosen
%                   pband; default:
%                   [2,inf] corresponding to frequency interval [0,pi]

```



```

%          and a particular series
%
%      .ga      : matrix with columns containing gain
%                between the ref. series and a particular series
%
%      .ph      : matrix with columns containing phase angle
%                between the ref. series and a particular series
%
%      .pht     : matrix with columns containing phase delay
%                of a particular series relative to the
%                ref. series
%
%      .phd     : matrix with columns containing group delay
%                of a particular series relative to the
%                ref. series
%
%      .mph     : vector with mean phase angle values
%                between the ref. series and a particular series
%
%      .fxconf  : confidence intervals for fx
%      .fyconf  : confidence intervals for fy
%      .cconf   : confidence intervals for co
%      .gconf   : confidence intervals for ga
%      .pconf   : confidence intervals for ph
%      .mphconf : confidence intervals for mph
%
%      All possible fields for ny > 2, if value for 'mulvar' is 1:
%
%      .frq     : frequencies
%
%      .fx      : (smoothed) periodogram of the reference series
%
%      .fy      : (smoothed) periodograms of all other series
%
%      .mco     : matrix with columns containing multiple coherency
%                between the ref. series and a particular series
%
%      .msco    : matrix with columns containing multiple
%                coherence (squared multiple coherency) between
%                the ref. series and a particular series
%
%      .pco     : matrix with columns containing partial coherency
%                between the ref. series and a particular series
%
%      .psco    : matrix with columns containing partial
%                coherence (squared partial coherency) between
%                the ref. series and a particular series
%
%      .pga     : matrix with columns containing partial gain
%                between the ref. series and a particular series
%
%      .pph     : matrix with columns containing partial phase angle
%                between the ref. series and a particular series
%
%      .ppht    : matrix with columns containing partial phase delay
%                of a particular series relative to the
%                ref. series
%
%      .pphd    : matrix with columns containing partial group delay
%                of a particular series relative to the
%                ref. series
%
%      .mpph    : vector with mean partial phase angle values
%                between the ref. series and a particular series
%
%      .fxconf  : confidence intervals for fx
%      .fyconf  : confidence intervals for fy
%      .mpphconf : confidence intervals for mpph
%
%      Examples :
%
%

```

```
%      s = spectran(y,per,'vnames',{ 'GDP','IPI' },'conf',1,'graphconf',1)
%      s = spectran(y,per,'pband',[8,32],'save',1)
%      s = spectran(y,per,'alpha',0.01,'phmean',1,'graph',0)
```

It is to be noted that the function **spectran** internally determines the name of the output files. If **spectran** has been called in a script file, the name of the output file (with either spectral analysis or lead-lag analysis results) contains the name of this script. The name of the output file is also denoted by a number that allows for distinction between files associated with different **spectran** calls. If at the beginning of each script run all existing variables are removed from the base workspace (but this does not happen between several possible **spectran** calls within the same script run), then the output files will be identified with the number of the corresponding **spectran** call. In case **spectran** is called from the command line or while evaluating a cell in the script file, the output files corresponding to spectral analysis become the name *spectran* and those referring to the lead-lag analysis – the name *leadlag*. For both types of files, also in this case a number is additionally appended to the file name. The number depends on how many structures from **spectran** calls already exist in the base workspace.

5 Examples

5.1 Example 1

The first example deals with quarterly cycles of the German industrial production index (IPI) and the consumer real wage in the time span 1970.Q1 – 2011.Q3. The data is stored in the files *PRGer.dat* and *CWGer.dat* in the subdirectory *data*. The demo file corresponding to this example is *exGER.m*. In the following, the discussion of the script files will be constrained to the selected code lines. The code below illustrates the estimation of spectra of the IPI cycle and the consumer real wage cycle.

```
% Settings for the spectral analysis:
alpha = 0.05; % compute conf. intervals with the significance level alpha
win = 1;      % Blackman–Tukey window
winlag = [];  % default window lag size

% Spectral analysis of the IPI
spx = periodg(x,win,winlag,alpha);

% Spectral analysis of the real wage
spy = periodg(y,win,winlag,alpha);
```

The following lines show how the estimated IPI spectrum can be written to a text file.

```

per = 4;          % frequency of the data
% frequency band for which the results are displayed;
% it corresponds to business cycle periodicities (periods between 6 and 32
% quarters)
fband = 2*pi./[32 6];
% IPI
fid = fopen('results\specPRGer.txt','w');
specwrite(fid,per,spx,fband,[],[],[],'PRGer')
fclose(fid);

```

Finally, with the code below the estimated IPI spectrum is plotted.

```

frq = spx.frq;
mul = 0;
fx = spx.f;
fxconf = spx.fconf;
specplot(frq,per,mul,fx,fband,fxconf,alpha,'Spectrum','PRGer')

```

5.2 Example 2

In the second example, three monthly US series between 1953.M4 and 2007.M9 are considered: the cycles of the industrial production index (IPI), consumption and working hours. The IPI cycle is here considered as the reference series and acts as the business cycle indicator. The code for this example can be found in the script file *exUS1.m* and the corresponding data in the file *DataUS.xls*. The following lines show how to compute the bivariate cross-spectral measures and how to allow for computing confidence intervals.

```

[ser,headers] = xlsread('data\DataUS.xls');
fband = 2*pi./[96 18];
vnames = headers;
% Compute confidence intervals with the significance level alpha
alpha = 0.05;
% default window function (Parzen window) and window lag size
cross = crossspan(ser,[],[],alpha);

```

In the next step, the mean phase angle values with their confidence intervals are calculated. The phase angle values are thereby averaged over the frequency band corresponding to the business cycle periodicities.

```

ph = cross.ph;
frq = cross.frq;
pband = [18,96];
[mph,mphconf] = meanphconf(ph,frq,pband);

```

The code below shows how to establish the lead-lag relation between the IPI cycle and the other series.

```
fid = fopen('results\leadlagUS.txt','w');
leadlagan(fid,per,cross,fband,mph,mphconf,fband,vnames,ppi)
fclose(fid);
```

Plots of the estimated phase angle values with their confidence bounds can be created in the following way:

```
phconf = cross.pconf;
phplot(frq,per,ph,fband,phconf,alpha,vnames)
```

The following code lines demonstrate how to calculate the multiple and the partial coherence (coherency), the partial phase angle and the partial gain between the IPI cycle and the other series. In addition, it is also shown how to write the results to a text file.

```
mulsp = mulparspan(ser,[],[],alpha);
fid = fopen('results\mulparUS.txt','w');
specwrite(fid,per,mulsp,fband,[],[],[],headers)
fclose(fid);
```

Using the output of the partial analysis, the partial phase angle can be plotted as follows:

```
parph = mulsp.pph;
phplot(frq,per,parph,fband,[],[],vnames)
```

5.3 Example 3

The third example refers to the demo file *exUS2.m* and to the same data set as used in the previous subsection. The purpose of this example is to present an alternative way to produce output similar to the one obtained with the script *exUS1.m*.

```
[ser,headers] = xlsread('data\DataUS.xls');
% Settings for spectran:
per = 12; % frequency of the data
pband = [18,96]; % Business cycle periodicities (expressed in months)
path = 'results';
% Other parameters are set to their default values
specb = spectran(ser,per,'vnames',headers,'corlag',12,'conf',1,...
    'pband',pband,'phmean',1,'phpband',pband,'graph',1,'graphconf',1,...
    'path',path,'leadlag',1,'pi',1);
```


To conduct the partial analysis instead, the value of the option “mulvar” must only be set to one, as is shown below:

```
specp = spectran(ser,per,'vnames',headers,'corlag',12,'conf',1,...  
    'pband',pband,'phmean',1,'phpband',pband,'graph',1,'graphconf',1,...  
    'path',path, 'leadlag',1,'pi',1,'mulvar',1);
```

References

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